

Singularity in Gravitational Collapse of Plane Symmetric Charged Vaidya Spacetime

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Abstract

We study the final outcome of gravitational collapse resulting from the plane symmetric charged Vaidya spacetime. Using the field equations, we show that the weak energy condition is always satisfied by collapsing fluid. It is found that the singularity formed is naked. The strength of singularity is also investigated by using Nolan's method. This turns out to be a strong curvature singularity in Tipler's sense and hence provides a counter example to the cosmic censorship hypothesis.

Keywords: Gravitational collapse; Naked singularity.

Oppenheimer and Snyder are the pioneers for the description of gravitational collapse of stars [1]. The study of gravitational collapse is motivated by the fact that it represents one of the few observable phenomena in the universe. The end state of a continual gravitational collapse of a massive star is an important issue in gravitation theory. According to Penrose, gravitational collapse of a star gives rise to a spacetime singularity [2] provided the conditions such as trapped surface formation etc are satisfied. Also, the singularity theorems of Hawking and Penrose provide a strong reason to believe that a singularity occurs at the end of gravitational collapse [3]. The

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spacetime singularity is a region where curvature and densities are infinite and their physical description is not possible. These singularities are of two kinds: naked if it is visible otherwise it is a clothed singularity, i.e., a black hole.

It would be interesting to investigate whether the singularity forming at the end of gravitational collapse is observable. There is an important conjecture related to the singularities known as cosmic censorship hypothesis (CCH) given by Penrose [4]. This states that the collapse of a physically reasonable initial data yields a spacetime singularity which is always hidden behind the event horizon. It has two versions, i.e., weak and strong. According to the weak version, singularity formed by gravitational collapse is not visible to a far away observer. The strong cosmic censorship hypothesis states that the singularity cannot be observed even by an observer who is very close to it. Wald [5] discussed some examples to justify the validity of weak form of CCH.

Despite of several attempts, there is no proof available for CCH and it remained an open problem. However, significant progress has been made in trying to find counter examples to CCH. Papapetrou [6] was the first who showed that the Vaidya solution [7] could give rise to naked singularities. This solution is widely used for discussing counter examples to CCH. Using the concept of gravitational lensing (GL), Virbhadra et al. [8] introduced a new tool for examining naked singularities. Gravitational lensing is the process of bending of light around a massive object such as a black hole. Virbhadra and Ellis [9] discussed GL by the Schwarzschild black hole. It was found that the relativistic images guarantee the Schwarzschild geometry close to event horizon. The same authors [10] also analyzed GL by a naked singularity. Claudel et al. [11] proved that the necessary and sufficient condition for the black hole to be surrounded by a photon sphere is that a reasonable energy condition holds. Virbhadra and Keeton [12] showed that weak CCH can be examined observationally without any uncertainty. Virbhadra [13] found that Seifert's conjecture is supported by the naked singularities forming during Vaidya null dust collapse. The same author developed an improved form of CCH using GL phenomenon [14].

Lemos [15] showed that the gravitational collapse of a spherical matter distribution in an anti-de Sitter spacetime form naked singularities, violating CCH while for the cylindrical or planar case, the collapse form black holes rather than naked singularities supporting CCH. Ghosh [16] introduced charged null fluid in the results found by Lemos. Harko and Cheng [17]

showed that a naked singularity is found in collapse of strange quark matter with Vaidya geometry for a particular choice of parameters. Sharif and his collaborators [18]-[23] discussed gravitational collapse in a variety of papers and also the effects of electromagnetic field are analysed.

Joshi and his collaborators [24]-[25] used dust collapse models to discuss physical features that caused the occurrence of naked singularities. The results are generalized using type *I* matter. They concluded that shearing forces and inhomogeneity within the collapsing matter lead to the formation of naked singularities. Zade et al. [26] found that the singularity is naked in monopole charged Vaidya spacetime.

In this brief paper, we analyze the singularity occurring in plane symmetric charged Vaidya spacetime. For this purpose, the spacetime for imploding radiations is given by [27]

$$ds^2 = \left(\frac{2m(v)}{r} - \frac{e^2(v)}{r^2} \right) dv^2 + 2drdv + r^2(dx^2 + dy^2). \quad (1)$$

Here $-\infty < x, y < \infty$ which describe 2-dimensional space and has topology $R \times R$, $-\infty < v < \infty$ is null coordinate, called the advanced Eddington time and $0 \leq r < \infty$ is the radial coordinate. This represents solution of the Einstein-Maxwell field equations with plane symmetry. We number the coordinates as $x^0 = v$, $x^1 = x$, $x^2 = y$ and $x^3 = r$.

We take the matter as charged null dust for which the energy-momentum tensor is [16]

$$T_{ab} = \rho \ell_a \ell_b + T_{ab}^{em}, \quad (2)$$

where the part $\rho \ell_a \ell_b$ is for null dust, ρ is the energy density of the null dust and ℓ_a is a null vector defined as

$$\ell_a = \delta_a^0, \quad \ell_a \ell^a = 0. \quad (3)$$

The electromagnetic energy-momentum tensor is

$$T_{ab}^{em} = \frac{1}{4\pi} [F_{ac} F_b^c - \frac{1}{4} g_{ab} F_{cd} F^{cd}], \quad (4)$$

where F_{ab} is the electromagnetic field tensor given by

$$F_{ab} = \frac{e(v)}{r^2} (\delta_a^0 \delta_b^3 - \delta_b^0 \delta_a^3). \quad (5)$$

Using the Einstein field equations, $G_{ab} = \kappa T_{ab}$, we obtain

$$\rho = \frac{1}{4\pi r^3}(r\dot{m} - e\dot{e}), \quad (6)$$

where dot denotes differentiation with respect to v . The weak energy condition is satisfied if $\rho \geq 0$, which is possible only when $r\dot{m} - e\dot{e} \geq 0$, i.e., $r \geq \frac{e\dot{e}}{\dot{m}}$. We know from the literature [28] that Lorentz force prevents the particle to move into the region where $r < \frac{e\dot{e}}{\dot{m}}$, so the energy condition is always satisfied.

It is assumed that the first wavefront arrives at $r = 0$ at time $v = 0$ and the final wavefront reaches the center at $v = T$. A singularity of growing mass develops here at $r = 0$. For $v < 0$, the spacetime is Minkowski with $m(v) = e(v) = 0$ and for $v > T$, the spacetime settles to plane symmetric Reissner Nordström solution. Thus, for $v = 0$ to $v = T$, the spacetime is plane symmetric charged Vaidya spacetime.

In order to check whether the singularity is naked or clothed, we examine the behavior of radial null geodesic. This is defined as [29]

$$ds^2 = 0, \quad dx = 0 = dy. \quad (7)$$

If the radial null geodesic equation admits at least one real and positive root then the singularity is naked. The geodesic equations for charged Vaidya spacetime, using Eq.(7), are given by

$$\ddot{v} + \left(\frac{mr - e^2}{r^3}\right)\dot{v}^2 = 0, \quad (8)$$

$$\ddot{r} + \left[\frac{\dot{m}r - e\dot{e}}{r^2} - \frac{(2mr - e^2)(mr - e^2)}{r^5}\right]\dot{v}^2 - 2\left(\frac{2mr - e^2}{r^3}\right)\dot{r}\dot{v} = 0. \quad (9)$$

Now $m(v)$ and $e(v)$ are unknown functions of v . For analytic solution, we take

$$m(v) = \frac{\lambda v}{2}, \quad e^2(v) = \mu v^2; \quad \lambda, \mu > 0. \quad (10)$$

Using these values in Eq.(1), it follows that

$$ds^2 = \left(\frac{\lambda v}{r} - \frac{\mu v^2}{r^2}\right)dv^2 + 2drdv + r^2(dx^2 + dy^2). \quad (11)$$

This represents a self-similar metric and $X = \frac{v}{r}$ is the self-similarity variable.

The radial null geodesics for this spacetime are given by

$$\frac{dv}{dr} = \frac{2}{\frac{-\lambda v}{r} + \frac{\mu v^2}{r^2}} \quad (12)$$

which has a singularity at $r = 0$, $v = 0$. Suppose that X_0 is the limiting value of X as the singularity is approached, i.e.,

$$X_0 = \lim_{r \rightarrow 0, v \rightarrow 0} X = \lim_{r \rightarrow 0, v \rightarrow 0} \frac{v}{r} = \lim_{r \rightarrow 0, v \rightarrow 0} \frac{dv}{dr}. \quad (13)$$

Using Eq.(12), this equation becomes

$$\mu X_0^3 - \lambda X_0^2 - 2 = 0. \quad (14)$$

We know that [30] *every equation of odd degree has at least one real root whose sign is opposite to that of its last term, the coefficient of first term being positive*. This implies that Eq.(14) has at least one real and positive root independent of the values of parameters λ and μ . Hence the singularity occurring in plane symmetric charged Vaidya spacetime is at least locally naked, in particular, for $\lambda = 0.1$, $\mu = 0.1$, $X_0 = 3.09198$. Singularity arising in uncharged case can be examined by taking $\mu = 0$ in Eq.(14). In this case, it turns out that Eq.(14) has no real root implying that the collapse ends at a black plane as final state.

To see whether it is a scalar polynomial singularity or not, we find the non-vanishing components of the Riemann tensor

$$\begin{aligned} R_{0113} &= -\left(\frac{-mr + e^2}{r^2}\right) = R_{0223}, \\ R_{1212} &= 2mr - e^2, \quad R_{0303} = \frac{-2mr + 3e^2}{r^4}, \\ R_{0101} &= -\frac{1}{r^4}(-r^4\dot{m} + r^3e\dot{e} + 2m^2r^2 - 3mre^2 + e^4). \end{aligned}$$

Using these values, the Kretschman scalar $\mathcal{R} = R_{abcd}R^{abcd}$, becomes

$$\mathcal{R} = \frac{4}{r^6}(4m^2 + \frac{e^4}{r^2} - \frac{4me^2}{r}). \quad (15)$$

Also in view of Eq.(10) and $X = \frac{v}{r}$, it takes the form

$$\mathcal{R} = \frac{4}{r^4}(\lambda^2 X^2 + \mu^2 X^4 - 2\lambda\mu X^3). \quad (16)$$

This shows that the Kretschman scalar diverges at the singularity and hence singularity is a scalar polynomial.

Now we explore the strength of singularity using Nolan's method [31] according to which a singularity is strong at $r = 0$ if \dot{r} has zero or infinite limit along every causal geodesic approaching the singularity. The strength of singularity is important in the sense that CCH may not be ruled out for weak naked singularities. We assume that \dot{r} is non-zero and finite as the singularity is approached, i.e.,

$$\dot{r} \sim h_0 \Rightarrow r \sim kh_0 \quad (17)$$

and

$$\frac{\dot{v}}{\dot{r}} = X_0 \Rightarrow v \sim kh_0 X_0, \quad (18)$$

here dot denotes differentiation with respect to affine parameter k . Using Eqs.(17) and (18), Eq.(8) implies that

$$\ddot{v} = Ck^{-1}, \quad C = -\left(\frac{\lambda}{2} - \mu X_0\right)h_0 X_0^3. \quad (19)$$

Since $k = 0$ as the singularity is approached which implies that \ddot{v} is undefined and hence \dot{v} is undefined. However, we already have $\dot{v} = h_0 X_0$. These values are consistent only if $C = 0$ giving rise to

$$X_0 = \frac{\lambda}{2\mu}. \quad (20)$$

Replacing this value of X_0 in Eq.(14), we get

$$\lambda^3 + 16\mu^2 = 0 \quad (21)$$

which is not possible as λ and μ are positive. Thus the inconsistency of Eqs.(18) and (19) imply that our supposition is wrong, i.e., \dot{r} is either zero or infinite. Hence singularity is strong in Tipler [32] sense providing a counter example to weak form of CCH.

Finally, we conclude that the singularity is at least locally naked violating weak CCH. The existence of positive roots of Eq.(14) only show that the null geodesics are coming out from the singularity but nothing could be said about the escape of these geodesics from the boundary of the collapsing matter. However, the mass function can be chosen in such a way that a locally naked singularity becomes globally naked [33].

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